$$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} \qquad f(x) = 3^x$$

$f(\text{bubba}) = 3^{\text{bubba}}$

deriv of f at $x=-1$

$$= \lim_{h \to 0} \frac{3^{(-1+h)} - 3^{(-1)}}{h}$$

use exponential rule

$A^{B+C} = A^B A^C$

Can factor $(3^{-1}) = \frac{1}{3}$.

useful fact

$3 = e^{\ln 3}$

$g(u) = u - \frac{1}{u} \overset{\text{show}}{\Longrightarrow} \frac{dg}{du}(c) = \frac{c^2+1}{c^2}$

Same as: If $g(x) = x - \frac{1}{x}$

Show that $g'(x) = \frac{x^2+1}{x^2}$.

Shortcut Methods for finding derivative:

① $(\sin(x))' = \cos(x)$

② Let's derive the <u>power rule</u>:

If n is a positive integer, then

$$(x^n)' = nx^{n-1}.$$

<u>Proof:</u> $f(x) = x^n$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{((x+h)^n) - (x^n)}{h}$$

<u>Side calculation:</u>

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$(x+h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

$$(x+h)^n = x^n + nx^{n-1}h + (\text{blah})h^2 + (\text{blahh})h^3 + \dots + h^n$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + (\text{blah})h^2 + (\text{blahh})h^3 + \dots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{nx^{n-1}h + (\text{blah})h^2 + (\text{blahh})h^3 + \dots + h^n}{h}$$

$\to 0$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(nx^{n-1} + (\text{blah})\,\overset{\to 0}{h} + (\text{blah})\,\overset{\to 0}{h^2} + \cdots + h^{n-1}\right)}{\cancel{h}}$$

$$= \boxed{nx^{n-1}}. \quad \text{QED} \doteq \blacksquare$$

2b) $(x^n)' = nx^{n-1}$ is true

if n is <u>any</u> real #.

<u>examples</u>: * $(x^3)' = \boxed{3x^2}$

** $(\sqrt{x})' = (x^{1/2})' = \frac{1}{2}x^{\left(\frac{1}{2}-1\right)}$

$$= \frac{1}{2} \cdot x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$$
